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Tutorial 5b - Exhaustible Resources - Additional Exercise

Let us suppose a continuum of oil markets identified by a time-index $t \in [0,+\infty)$. At each date, the flow of extraction and the reserves remaining unexploited are denoted respectively by $R(t)$ and $S(t)$, expressed in units of oil. Hence, $S(t) = S(0) - \int_0^t R(s)\,ds,\ S(0) = S_0,\ given$. There is no storage possibility and extraction is irreversible. Extraction is assumed to be free.

At any date, the inverse demand for oil by the households is given by $D^{-1}(R(t)) = \frac{1}{a(t)} R(t)^{-1/\alpha}$, where $a(t) > 0$ is an index of technical progress in the resource use rising at a rate $x > 0$ and $\alpha > 0$ is the price-elasticity of the demand for oil in absolute value. The social discount rate is $r > 0$.

A) Preliminary question

Express the instantaneous consumer surplus. Deduce the social surplus discounted at date $t = 0$.

B) Welfare Analysis

1) Write the social planner program. Briefly explain why we are allowed not to consider the non-negativity constraint on the extracted flow.
2) Write the present-value Hamiltonian function. Derive the first-order conditions of the above problem together with the associated transversality condition.
3) How evolve the present-value co-state variable? Interpret this law of motion in terms of intertemporal arbitrage.
4) Show from the transversality condition that the resource will be exhausted in the long run.
5) Find out the optimal pace of extraction.

C) Competitive equilibrium

1) Solve the intertemporal program of the extractor and the static one of the household.
2) Give and interpret the dynamics of the price. Derive the pace of extraction.
3) Express the price for the resource at any date t. What does it depend on?